A q-analogue for bi^{s}nomials coefficients and generalized Fibonacci sequence

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For $s \geq 1$, bi^{s}nomials coefficient denoted by $\binom{n}{k}_s$ are considered as extension of binomial coefficients $\binom{n}{k}$ and are obtained by the multinomial expansion (see [2])

$$\left(1 + x + x^2 + \cdots + x^s\right)^n = \sum_{k \geq 0} \binom{n}{k}_s x^k, \tag{1}$$

Andrew and baxter [1] defined a q-analogue for bi^{s}nomials coefficients by the q-binomials coefficients as follow

For $\alpha = 0, 1, ..., s$

$$\begin{bmatrix} n \\ k \end{bmatrix}_s^{(\alpha)} = \sum_{j_1 + j_2 + \cdots + j_s = k} \begin{bmatrix} n \\ j_1 \end{bmatrix} \begin{bmatrix} j_1 \\ j_2 \end{bmatrix} \cdots \begin{bmatrix} j_{s-1} \\ j_s \end{bmatrix} q^{\sum_{r=1}^{s-1}(n-j_r)j_{r+1} - \sum_{r=s-\alpha}^{s-1} j_{r+1}}.$$

Our communication will proceed according to the following steps;

We establish a new expression for the bi^{s}nomials coefficients.

According this expression we define a q-analogue of bi^{s}nomials coefficients $\begin{bmatrix} n \\ k \end{bmatrix}_q^{(s)}$.

With this new definition, we obtain a q-analogue of formula (1).

We suggest a generalized q-Fibonacci sequence which gives for $s = 1$ a Cigler's q-Fibonacci sequence [3].

References

[1] G. E. Andrews, J. Baxter, Lattice gas generalization of the hard hexagon model III q-trinomials coefficients, *J. Stat Phys.,* **47, (**1987), 297–330.

[2] H. Belbachir, S. Bouroubi, A. Khelladi, Connection between ordinary multinomials, Fibonacci numbers, Bell polynomials and discrete uniform distribution, *Annals Mathematicae et Informaticae,* **35,** *(*2008), 21–30.

[3] J. Cigler, A new class of $q-$Fibonacci polynomials, *Electronic J. Combinatorics* **10**, (2003), Article R19.