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Solutions to a ferrofluid flow with heat transfer

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Abstract

Ferrofluids are widely used in the areas of engineering and biomedical. In this work, we study a model describing the dynamic of a ferrofluid flow under the action of an applied magnetic field, taking into account the heat transfer. The problem comprises the Navier-Stokes equation for the velocity, the Bloch-Torrey equation for the magnetization and the magnetostatic equation for the magnetic field. To describe the heat transfer, we use a generalization of the usual equation given by the Maxwell-Cattaneo law which is a system satisfied by the temperature and the heat flux. For this system, we prove a global existence of weak solutions having a finite energy.

Résumé

Dans ce travail, on s'intéresse à un modèle d'écoulement d'un fluide magnétique soumis à un champ magnétique. Le système étudié comporte plusieurs inconnues: la vitesse du fluide décrite par l'équation de Navier-Stokes incompressible, la magnétisation décrite par l'équation de Bloch-Torrey, le champ magnétique vérifiant l'équation de la magnétostatique et comme on s'intéresse aussi dans ce modèle au transfert de la chaleur dans le fluide, le problème est complété par le système de Maxwell-Cattaneo qui décrit l'évolution de la température et du flux de chaleur. Un résultat d'existence globale d'une solution faible est démontré pour ce système.

AMS subject classifications: 76N10, 35Q35, 76D05.

Keywords: Navier-Stokes equations, Bloch-Torrey equation, magnetostatic equation, Maxwell-Cattaneo law, regularization, Galerkin' scheme, monotonicity, transport equations, parabolic equations

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